Homework 8: Limits of Functions

Assignments should be ***stapled*** *and written clearly and legibly.*

1. Let $A \subseteq B$ be two sets of real numbers. We say that A is **dense** in B if $\overline{A} = B$. Prove that $\mathbb{Q}$ is dense in $\mathbb{R}$. (In other words, prove $\overline{\mathbb{Q}} = \mathbb{R}$.)

2. §5.1, #7(a), #13 (the Squeeze Theorem).

3. For $\lim_{x \to 2}(x^2 + 2x + 3) = 11$, illustrate the $\epsilon - \delta$ definition of a limit by finding values of δ that correspond to $\epsilon = 1$ and $\epsilon = 0.1$.

4. Determine the following limits, and then use the $\epsilon - \delta$ definition of limit to prove your answers.
   (a) $\lim_{x \to 3} \frac{x^2 + 2x - 15}{x - 3}$
   (b) $\lim_{x \to 2} x^4$

5. Use the Squeeze Theorem to prove that $\lim_{x \to 0} \left(x \sin\left(\frac{1}{x}\right)\right) = 0$. Make sure to state what D is.

6. Let $f : D \to \mathbb{R}$ and let a be a limit point of D. Suppose that $\lim_{x \to a} f(x) > 0$. Prove that there exists a deleted neighborhood $N^*_\delta(a)$ of a such that $f(x) > 0$ for all $x \in N^*_\delta(a) \cap D$.