

## Homework 4: The Limit of a Sequence

Assignments should be **stapled** and written clearly and legibly.

1. §4.1, #6(c).
2. Using only the definition of a limit, prove

$$(a) \lim_{n \rightarrow \infty} \frac{2n - 2}{5n + 3} = \frac{2}{5}$$

$$(b) \lim_{n \rightarrow \infty} \left( 5 - \frac{1}{\sqrt{n + \sqrt{n + 12}}} \right) = 5$$

3. Consider the following definition:

*Definition:* A sequence  $(a_n)$  is said to **reverge** to  $L$  if there exists  $\epsilon > 0$  such that for every  $N \in \mathbb{N}$ , whenever  $n \geq N$ , we have  $|a_n - L| < \epsilon$ .

- (a) Give an example of a sequence that reverges.
  - (b) If a sequence reverges, must it also converge? If not, give a counterexample.
  - (c) Is it possible for a sequence to reverge to two different values?
  - (d) (optional) If possible, give a simpler definition of revergence.
4. Does the sequence  $(a_n)$  defined by

$$a_n = \begin{cases} 2, & \text{if } n = 2^m \text{ for some } m \in \mathbb{N} \\ 1 + \frac{1}{n}, & \text{otherwise} \end{cases}$$

converge? Justify your answer. (A formal proof is not required.)

5. Suppose that a sequence  $(a_n)$  converges to 0, and that at least one term  $a_n$  of the sequence is greater than 0. Prove that the set  $\{a_n\}$  has a maximum.