Homework 2: Functions, More Supremums

*Assignments should be **stapled** and written clearly and legibly.*

1. §2.3, #5(a), (b), 7(b), (c), (e), 8.
2. Prove that $\sup(-\infty, 3) = 3$.
3. Let $a < b$ be real numbers and consider the set $T = \mathbb{Q} \cap [a, b]$. Prove that $\sup T = b$. You may use any of the theorems we've proved in class.
4. Suppose that A and B are nonempty bounded sets in $\mathbb{R}$ and that $\sup A < \sup B$. Prove that there exists $b \in B$ that is an upper bound for A. Then show by example that this is not always the case if we only assume $\sup A \leq \sup B$.
5. Let a and b be real numbers. Prove that if $a < b + \epsilon$ for every $\epsilon \in \mathbb{R}$ such that $\epsilon > 0$, then $a \leq b$.