

## Homework 12: The Mean Value Theorem

*Directions. Assignments should be **stapled** and written clearly and legibly.*

1. §6.2, #16.
2. Find a twice differentiable function  $f(x)$  such that  $f'(1) = -1$ ,  $f'(4) = 7$ , and  $f''(x) > 3$  for all  $x$ , or prove that such a function cannot exist.
3. Suppose that  $f : [a, b] \rightarrow \mathbb{R}$  has continuous derivatives  $f'$  and  $f''$ , and assume there exists  $c \in (a, b)$  such that  $f(a) = f(c) = f(b)$ . Prove that there exists  $d$  in  $(a, b)$  such that  $f''(d) = 0$ .  
Hint. First prove that there exist  $x_1$  and  $x_2$  such that  $a < x_1 < x_2 < b$  and  $f'(x_1) = f'(x_2) = 0$ .
4. A number  $c$  is called a **fixed point** of a function  $f$  if  $f(c) = c$ . Prove that if  $f$  is a differentiable function and  $f'(x) \neq 1$  for all real numbers  $x$ , then  $f$  has at most one fixed point.
5. Suppose that  $f$  and  $g$  are differentiable functions such that  $f' = g$  and  $g' = -f$ . Prove that  $h(x) = (f(x))^2 + (g(x))^2$  is a constant function. (Hint: Use the chain rule.)