Homework 10: Continuity and the Intermediate Value Theorem

Assignments should be **stapled** and written clearly and legibly.

- 1. $\S5.3, \#5, 7.$
- 2. Let f be continuous on [0,1] with f(0) = f(1). Prove that there exists $c \in [0,\frac{1}{2}]$ such that $f(c) = f(c + \frac{1}{2})$.
- 3. Prove that there exists a real number x such that

$$x^{177} + \frac{165}{1 + x^8 + \sin^2 x} = 125.$$

- 4. Suppose that f is continuous on [a, b] and for every $x \in [a, b]$, $f(x) \in \mathbb{Q}$. Prove that f is constant on [a, b]. (Hint: use proof by contradiction.)
- 5. Let $f : \mathbb{R} \to \mathbb{R}$ be continuous.
 - (a) Suppose that $D \subseteq \mathbb{R}$ is dense in \mathbb{R} , i.e., $\overline{D} = \mathbb{R}$. Prove that if f(x) = 0 for all $x \in D$, then f(x) = 0 for all $x \in \mathbb{R}$.
 - (b) Use part (a) and Problem 1 of Homework 8 to prove that if f(x) = 0 for all $x \in \mathbb{Q}$, then f(x) = 0 for all $x \in \mathbb{R}$.