Homework 4.5: Independence

1. Two dice, one red and one green, are rolled. Define the events

   A: the red die lands on a 3, 4, or 5

   B: the sum of the two dice is 9

   (a) Compute $P(A \mid B)$.

   (b) Compute $P(B \mid A)$.

   (c) Are A and B independent events? Justify your answer.

2. Recall that three events A, B, and C are *independent* if

$$\begin{aligned} P(A \cap B) &= P(A) \cdot P(B) \\ P(B \cap C) &= P(B) \cdot P(C) \\ P(A \cap C) &= P(A) \cdot P(C) \end{aligned} \tag{1}$$

   and

$$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C) \tag{2}$$

   (a) Two dice, one red and one green, are rolled. Define the events

   A: the red die lands on a 1, 2, or 3

   B: the red die lands on a 3, 4, or 5

   C: the sum of the two dice is 9

   Do these three events satisfy (1)? Do they satisfy (2)?

   (b) A roulette wheel has thirty-six numbers colored red or black according to the pattern indicated below:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
R	R	R	R	R	B	B	B	B	R	R	R	R	B	B	B	B	B
36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19

   Define the events

   A: red number appears

   B: even number appears

   C: number is less than or equal to 18

   Do these three events satisfy (1)? Do they satisfy (2)?