Homework 15: Hypergeometric, Geometric, and Negative Binomial Distributions

1. To demonstrate his benevolence, Darth Vader decides to use the Death Star to randomly destroy six of the twelve planets of the Dagobah system (rather than all twelve, as suggested by the Emperor). Four of the twelve planets harbor life.
   (a) Find the probability that at least three of the four planets harboring life are destroyed.
   (b) Find the expected number of planets harboring life that will be destroyed.

   For both parts, you should ignore the fact that Yoda lives in the Dagobah system.

2. Because of his past convictions for fraud, Chester has a 30% chance each year of having his tax returns audited. What is the probability that he will escape detection for at least three years. Assume that he commits fraud every year.

3. Mad Max is applying for a driver's license. Write the formula for $f_X(x)$, where X is the number of tries that he needs in order to pass the driving test. Find $E(X)$. Assume that his probability of passing the driving test on any given attempt is 0.10.

4. The Millenium Falcon can withstand four direct hits from an enemy destroyer, but no more. Suppose that an Imperial Navy Destroyer fires a barrage of missiles at the Falcon, each with a 20% chance of hitting. Find the probability that the twelfth missile destroys the Falcon.

5. Suppose that Gertrude has a 70% free-throw percentage in basketball. On average, how many free-throws does she have to take in order to get 10 points, assuming that each free throw counts for 2 points.

6. Country A inadvertently launches ten guided missiles – six armed with nuclear warheads – at country B. In response, country B fires seven antiballistic missiles, each of which will destroy exactly one of the incoming rockets. The antiballistic missiles have no way of detecting, though, which of the ten rockets are carrying nuclear warheads.
   (a) Find the probability that country B will be hit by at least two nuclear missiles.
   (b) Find the expected number of nuclear missiles that will hit country B.

7. A fair coin is flipped until the first head appears. You win \$2 if the first head appears on the first toss, \$4 if it appears on the second toss, and in general, \$$2^k$ if it appears on the k-th toss. Find your expected winnings. (This problem is known as the *St. Petersburg paradox*. It was studied by Daniel Bernoulli in 1738.)