

## Homework 12: Indicator Random Variables

1. You randomly throw 6 balls into 10 different baskets. Let  $X$  be the number of balls which land in the first basket, and let  $Y$  be the number of baskets which are empty.
  - (a) Find  $f_X(x)$ .
  - (b) Find  $E(X)$ .
  - (c) Find  $E(Y)$ . (Hint. Let  $I_j$  be the indicator of the event “basket  $j$  is empty”. Note that  $Y$  is the sum of the  $I_j$ 's.)
2. From a group of 8 math majors and 7 physics majors (no double majors), 4 are randomly selected for the Putnam Competition team. Let  $M$  be the number of math majors on the team. Find  $E(M)$  in two ways:
  - (a) Use the definition of  $E(M)$ . (You will need to first find  $f_M(m)$ .)
  - (b) Use indicator random variables. Specifically, let  $I_j$  be the indicator of the event “the  $j$ -th person selected is a math major”. Use the fact that  $M = I_1 + I_2 + I_3 + I_4$ .
3. A population of  $n$  people vote in an election.  $d$  vote democratic and  $n-d$  vote republican. In the next election, the probability of a democratic voter switching to republican is  $p_1$ , and the probability of a republican voter switching to democratic is  $p_2$ . Let  $X$  be the number of democratic votes in the second election. Find  $E(X)$ .