

## Homework 11: Expectation of a Function of a Random Variable

1. Let  $X$  have pdf

$$f_X(x) = 2(1 - x), 0 \leq x \leq 1$$

Suppose that  $Y = X^2$ . Find  $E(Y)$  in two different ways.

2. A box is to be constructed so that its height is five inches and its base is  $X$  inches by  $X$  inches, where  $X$  is a random variable with pdf  $f_X(x) = 6x(1 - x)$ ,  $0 < x < 1$ . Let  $V$  be the volume of the box. Find  $E(V)$ .
3. Suppose that the hypotenuse of an isosceles right triangle is a random variable which is uniform over the interval  $[4, 10]$ . Find the expected value of the triangle's area.
4. Grades on a recent math exam were fairly low. When graphed, the distribution of grades had a shape similar to the pdf

$$f_X(x) = \frac{1}{5000}(100 - x), 0 \leq x \leq 100$$

In order to “curve” the results, the professor announces that she will replace each person's grade  $X$  with a new grade  $g(X)$ , where  $g(X) = 10\sqrt{X}$ . Will this curve raise the class average above 60?

5. Suppose that  $Y$  has pdf

$$f_Y(y) = \frac{1}{\pi(1 + y^2)},$$

for  $-\infty < y < \infty$ . Let  $Z = 1/(1 + Y^2)$ . Then  $Z$  has a pdf of the form  $cz^\alpha(1 - z)^\beta$ ,  $0 < z < 1$ .

- (a) Find  $c$ ,  $\alpha$ , and  $\beta$ .
- (b) Find  $E(Z)$ .