Homework 16: Inner Product Spaces

Assignments should be ***stapled*** *and written clearly and legibly.*

1. §6.1, #34, 36, 38, 44. In problem 34, for each axiom which does not hold, give a counterexample demonstrating this.

2. §6.2, #39, 40.

3. Let $\mathbf{u} = \left[\begin{smallmatrix} u_1 \\ u_2 \end{smallmatrix}\right]$ and $\mathbf{v} = \left[\begin{smallmatrix} v_1 \\ v_2 \end{smallmatrix}\right]$. Show that $\langle \mathbf{u}, \mathbf{v} \rangle = 3u_1v_1$ does not define an innter product on $\mathbb{R}^2$. List all inner product axioms that fail to hold. For each axiom which does not hold, give a counterexample demonstrating this.

4. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be rotation by θ about the line through $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ (counterclockwise, as viewed from the tip of $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$). Let $\mathcal{B}$ be the the following basis of $\mathbb{R}^3$:

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \right\}.$$

   Let $\mathcal{B}' = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$, the standard basis for $\mathbb{R}^3$.

   (a) Confirm that $\mathcal{B}$ is an orthonormal basis for $\mathbb{R}^3$.

   (b) Find $[T]_{\mathcal{B}}$.

   (c) Use the change of basis formula to find $[T]_{\mathcal{B}'}$, the standard matrix for T.

   (d) Using (c), find the rotation of $\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ by $\pi/3$ about the line through $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.

   Note. For this problem, you should use the Euclidean inner product (i.e., the dot product) on $\mathbb{R}^3$.