Homework 15: The Change of Basis Formula

*Assignments should be **stapled** and written clearly and legibly.*

1. §8.5, #10, 13.

2. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be projection to the line $y = \frac{1}{5}x$. Let $\mathcal{B} = \left\{ \left[\begin{smallmatrix} 5 \\ 1 \end{smallmatrix}\right], \left[\begin{smallmatrix} -1 \\ 5 \end{smallmatrix}\right] \right\}$ and $\mathcal{C} = \{\mathbf{e}_1, \mathbf{e}_2\}$, two bases of $\mathbb{R}^2$.
   (a) Find $[T]_{\mathcal{B}}$.
   (b) Use (a) and the change of basis formula to find $[T]_{\mathcal{C}}$, the standard matrix for T.
   (c) Find the projection of $\left[\begin{smallmatrix} 3 \\ 7 \end{smallmatrix}\right]$ to the line $y = \frac{1}{5}x$, i.e., find $T\left(\left[\begin{smallmatrix} 3 \\ 7 \end{smallmatrix}\right]\right)$.

3. Let $T : P_1 \to P_1$ be the linear transformation given by $T(p(x)) = xp'(x) + p(x)$. Let $\mathcal{B} = \{1, x\}$ and $\mathcal{C} = \{1 + 2x, 3 + 5x\}$, two bases for P_1.
   (a) Find the change of coordinates matrices $[I]_{\mathcal{B},\mathcal{C}}$ and $[I]_{\mathcal{C},\mathcal{B}}$.
   (b) Find $[T]_{\mathcal{B}}$.
   (c) Use the change of basis formula and your answers to parts (a) and (b) to find $[T]_{\mathcal{C}}$.

4. Let a_1, a_2, a_3 be constants. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear operator defined by the formula
$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} a_1x_1 \\ a_2x_2 \\ a_3x_3 \end{bmatrix}$$
   (a) Under what conditions will T have an inverse?
   (b) Assuming the conditions determined in part (a) are satisfied, find a formula for $T^{-1}\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right)$.