Homework 12: Compositions, Inverse Transformations, Isomorphisms

*Assignments should be **stapled** and written clearly and legibly. Problem 6 is optional.*

1. §8.2, #20, 26.

2. §8.3, #2, 4, 10(b). (Please provide justifications.)

3. Consider $T : P_2 \to P_2$ defined by $T(p(x)) = p'(x)$, where $p'(x)$ is the derivative of $p(x)$. Prove that T is linear. Then determine $\ker(T)$ and $R(T)$. Verify that Theorem 2.5 holds for T.

4. Consider the linear transformation $T : P_3 \to P_3$ given by $T(p(x)) = xp'(x)$. Determine $\ker(T)$ and $R(T)$.

5. Consider $T : P_2 \to P_2$ given by $T(p(x)) = p(x) + p'(x)$.
   (a) Prove that T is linear.
   (b) Prove that T is an isomorphism.
   (c) Give $\ker(T)$ and $R(T)$.

6. (Optional) Let $T : V \to W$ be a linear transformation, and let Y be a subspace of W. The **inverse image** of Y, denoted $T^{-1}(Y)$, is defined to be

$$T^{-1}(Y) = \{\mathbf{v} \in V : T(\mathbf{v}) \in Y\}$$

   Prove that $T^{-1}(Y)$ is a subspace of V.

   Note: The symbol T^{-1} is used to represent both the inverse image, as defined above, and the inverse transformation, as defined in class on Wednesday. Although the same symbol is used for both, they are different concepts.