Homework 11: One-to-One, Onto

Assignments should be ***stapled*** *and written clearly and legibly.*

1. §8.2, #1(c), 6, 7, 19(a)(c). Make sure to justify your answer to the question asked in Exercise 7.

2. Let $T\colon V \to W$ be linear transformation, and let $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ be linearly independent in V.

   (a) Prove that if T is one-to-one, then $\{T(\mathbf{v}_1), \dots, T(\mathbf{v}_p)\}$ is linearly independent in W.

   Hint. Begin the proof as follows:

   "Suppose $c_1 T(\mathbf{v}_1) + \cdots + c_p T(\mathbf{v}_p) = \mathbf{0}$. I must show that $c_1 = \cdots = c_p = 0$."

   (b) Give an example showing that if T is not one-to-one, then $\{T(\mathbf{v}_1), \dots, T(\mathbf{v}_p)\}$ need not be linearly independent in W.

3. Let $T\colon V \to W$ be a linear transformation, with $\dim V = n$, $\dim W = m$. Prove the following:

   (a) $\dim(R(T)) \leq n$.

   (b) $\dim(R(T)) = n$ if and only if T is one-to-one.

   (c) $\dim(R(T)) = m$ if and only if T is onto.

4. (GRE Mathematics Subject Test) Let V be the vector space of all 2×3 matrices, and let W be the vector space of all 4×1 column vectors. If T is a linear transformation from V onto W, what is the dimension of the subspace $\{\mathbf{v} \in V : T(\mathbf{v}) = \mathbf{0}\}$? (Circle the correct answer and explain your reasoning.)

   (A) 2   (B) 3   (C) 4   (D) 5   (E) 6