

Math 221: Homework Exercises

1 The Definition of a Derivative

- For each of the following functions, use limits to find (i) the slope of the tangent line at $x = 3$, and (ii) the equation of the tangent line at $x = 3$.
 - $f(x) = 3x^2 + 2x$.
 - $g(x) = \frac{5}{x}$.
- For each of the following functions, use limits to find (i) the slope of the tangent line at $x = 1$, and (ii) the equation of the tangent line at $x = 1$.
 - $g(x) = \frac{2}{x}$.
 - $f(x) = 5x^2$.
 - $h(x) = \sqrt{x}$.
- For each of the following functions, use limits to find (i) the slope of the tangent line at $x = 4$, and (ii) the equation of the tangent line at $x = 4$.
 - $f(x) = 5x^2 - 2x + 7$.
 - $g(x) = \frac{3}{x}$.
 - $h(x) = 3\sqrt{x}$.
- Use limits to find $f'(x)$ for (a) $f(x) = 1/x$, (b) $f(x) = 3x^2 - 2x - 5$, (c) $f(x) = \sqrt{x}$.
- An object is dropped from a tall building on planet Krypton. The distance the object falls in t seconds is $s(t) = 12t^2$ m.
 - Use limits to find $s'(t)$.
 - Find the object's velocity after 2 seconds.
 - How much time is required for the object's velocity to reach 36 m/s?
 - When the object's velocity is 36 m/s, how far has the object fallen?
 - If the height of the building is 300 m, what will be the object's velocity when it hits the ground?

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6. Let $f(x) = 3x^2 + 2x$.
- (a) Use limits to find $f'(x)$.
 - (b) Find the equation of the tangent line to the graph of $f(x)$ at $x = 1$.
 - (c) Find the point on the graph of $f(x)$ where the tangent line has slope 20.
 - (d) Find an equation of the tangent line to the graph of $f(x)$ which has slope 20.
7. An object travels along a straight line. Its position after t seconds is $s(t) = 12t - t^2$ meters.
- (a) Find the average velocity of the object over the time interval $[1, 3]$.
 - (b) Find the average velocity of the object over the time interval $[1, 1 + h]$. Simplify your answer as much as possible. (Assume h is not equal to 0).
 - (c) Use part (b) to find the instantaneous velocity of the object at time $t = 1$ second.